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CRACK TIP PLASTICITY IN DYNAMIC FRACTURE MECHANICS.(U)

APR 78 J D ACHENBACH, M F KANNINEN

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CRACK TIP PLASTICITY IN DYNAMIC  
FRACTURE MECHANICS

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The ultimate goal of the research reported in this paper is to provide the basis of a computational procedure for plastic dynamic crack propagation in structures of engineering interest. The prerequisite for such a development is knowledge of the nature of the crack tip singularity, which can be obtained via an asymptotic analysis in which attention is focused on the very near crack tip region. Here we have considered the specific case of crack propagation in anti-plane strain (Mode III). The results suggest some interesting general conclusions, and the analysis has pointed the way to the solution of the Mode I problem.

The material model used in this work is based on Prandtl-Reuss incremental plasticity with a bilinear stress-strain relation. Irreversible material unloading behind the crack tip is specifically allowed. This formulation leads to a set of three first order ordinary differential equations that are nonlinear with variable coefficients. Therefore, a numerical solution was necessary. The results show that  $s$ , the order of the crack tip singularity, and  $\theta_p$ , the angle defining the position of the plastic unloading interface, while highly dependent on the slope of the stress-strain curve in the plastic regime, are much less dependent on the crack speed. Specifically, for a given crack speed, both  $|s|$  and  $\theta_p$  increase with the slope of the plastic portion of the stress-strain curve. For a given bilinear relation,  $|s|$  decreases modestly with crack speed while  $\theta_p$  increases. Possibly of most significance, it appears that the change in the order of the singularity with crack speed may be considered to be negligible if the changes in the crack speed are not too large. If borne out by the analysis of the Mode I problem now in progress, this finding will greatly simplify the computation of dynamic crack propagation/arrest events.

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## CRACK TIP PLASTICITY IN DYNAMIC FRACTURE MECHANICS

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### INTRODUCTION AND SUMMARY

Dynamic fracture mechanics encompasses all problems involving crack growth initiation, propagation, and arrest in which, for an acceptable solution, inertia forces must be included in the equations of motion of the cracked body. At present, dynamic fracture mechanics solutions are largely confined to conditions where linear elastic fracture mechanics (LEFM) is valid. These are appropriate when the plastic deformation attending the crack tip is small enough to be dominated by the elastic field surrounding it. Problems of crack growth initiation under impact loads and of rapid unstable crack propagation and crack arrest can be treated with LEFM by using dynamically computed stress intensity factors and experimentally determined dynamic fracture toughness values. However, as in static conditions, there are many tough, ductile materials for which LEFM cannot be confidently applied. Currently, there is little alternative: a dynamic plastic fracture methodology does not now exist. Indeed, work in developing a plastic fracture mechanics treatment of the slow stable crack growth under quasi-static monotonically increased loading has not yet come to maturity.

The objective of this work is to develop a procedure by which crack tip plasticity can be taken directly into account in rapid crack propagation. To set the stage, a background description of linear elastic dynamic fracture mechanics is first given. Then, existing solutions for dynamic crack propagation and for quasi-static crack growth accompanied by crack tip plasticity are reviewed. It is found that existing dynamic plastic fracture solutions are essentially confined to strip yield (Dugdale model) plastic zones that are collinear with the crack. In addition, such models do not contain the effect of material unloading. It has been concluded that more realistic treatments of crack tip plasticity via an incremental plasticity formulation for a propagating crack are needed.

The ultimate goal of the research reported in this paper is to provide the basis of a computational procedure for plastic dynamic crack propagation in structures of engineering interest. We envision that the results might be used to construct a special crack tip element in a finite element or other numerical analysis procedure. The prerequisite for such a development is knowledge of the nature of the crack tip singularity. This can be obtained via an asymptotic analysis in which attention is focused on the very near crack tip region. Previous solutions for a crack propagating dynamically with attendant plastic deformation have not been able to include

the singularity. This has been accomplished here with an asymptotic analysis in the specific case of antiplane shear (Mode III) crack propagation. While such calculations have little practical significance, the solution presented in this paper has allowed some important conclusions to be drawn and has pointed the way to the solution of Mode I problems.

In an asymptotic analysis, only the highly strained material in the near tip regime is considered. Hence, it is the stress-strain behavior at very large strains that is important. In fact, the limiting speed for crack propagation is dictated by the slope of the stress-strain curve at large strain. In this sense only a material model with a finite slope of the stress-strain curve at large strains offers a basis for such calculations; i.e., any other formulation will give either a zero or an infinite wave speed for large strain, neither of which are physically realistic.

The material model used in this work is based on Prandtl-Reuss incremental plasticity with a bilinear stress-strain relation. Irreversible material unloading behind the crack tip is specifically allowed. This formulation leads to a set of three first order ordinary differential equations that are nonlinear with variable coefficients. Therefore, a numerical solution was necessary. The results show that  $s$ , the order of the crack tip singularity, and  $\theta_p$ , the angle defining the position of the plastic unloading interface, while highly dependent on the slope of the stress-strain curve in the plastic regime, are much less dependent on the crack speed. Specifically, for a given crack speed, both  $|s|$  and  $\theta_p$  increase with the slope of the plastic portion of the stress-strain curve. For a given bilinear relation,  $|s|$  decreases modestly with crack speed while  $\theta_p$  increases. Possibly of most significance, it appears that the change in the order of the singularity with crack speed may be considered to be negligible if the changes in the crack speed are not too large. If borne out by the analysis of the Mode I problem now in progress, this finding will greatly simplify the computation of dynamic crack propagation/arrest events.

#### STATUS OF DYNAMIC FRACTURE MECHANICS

There are two generic problems that fall into the domain of dynamic fracture mechanics. These are, (1) a cracked body subjected to a rapidly varying loading, and (2) a body containing a rapidly propagating crack. The first type has wide applicability. Several laboratory test specimens (e.g., Charpy, Drop Weight Tear Test) and virtually all structural components subjected to impact loading fall into this category. The second type of problem, while having a much narrower field of applicability, is no less important. There are several kinds of engineering structures in which unchecked unstable crack growth would have catastrophic consequences. These include gas transmission pipelines, ship hulls, and nuclear reactor components. In these structures, it is essential to go beyond the normal fracture mechanics design philosophy of simply attempting to preclude crack growth initiation. Specific attention must be given to the arrest of unstable crack propagation. This second line of defense requires direct consideration of rapid crack propagation preceding arrest.

Engineering structures requiring protection against the possibility of large-scale catastrophic crack propagation are generally constructed of ductile, tough materials. For the initiation of crack growth, LEFM procedures can give only approximately correct predictions for such materials. The elastic-plastic treatments required to give precise results have not yet been developed in a completely acceptable manner, even under static conditions. The following describes current progress in this area to provide a starting point for the development of the dynamic plastic propagating crack tip analysis that is the objective of this work.



# Linear Elastodynamic Treatments

Under the assumption of linear elastic material behavior, the most prominent parameter is the elastodynamic stress intensity factor. This parameter, which enters in the computed elastodynamic stress field in the immediate vicinity of a crack tip, depends on time  $t$  and on the speed of the crack tip,  $v$ . It is given the symbol  $k = k(t, v)$  to distinguish it from the stress intensity factor for the corresponding quasi-static problem (when inertia terms are ignored in the computations), indicated by  $K = K(t)$ . Although not explicitly indicated, both  $k(t, v)$  and  $K(t)$  also depend on the crack length, on the external geometry of the cracked body, on material constants, and of course on the external loads.

The general form of the elastodynamic near-tip fields for a crack propagating rapidly along a rather arbitrary but smooth trajectory is well known. Let us consider a crack propagating in its own plane with a time-varying crack-tip speed  $v(t)$ . The two-dimensional geometry with a system of moving polar coordinates centered at the crack tip is shown in Fig. 1.

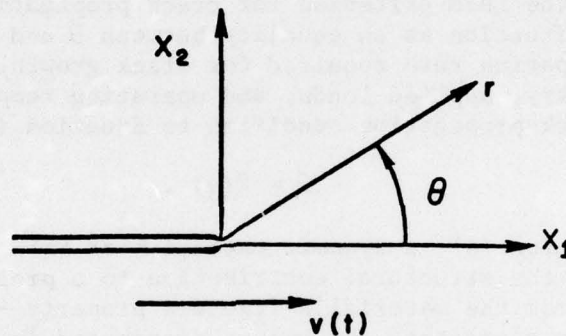


Fig. 1

For symmetric opening up of the crack (Mode I), the instantaneous hoop stress near the crack tip may be expressed as

$$\tau_{\theta} \sim \frac{1}{(2\pi)^{1/2}} \frac{1}{r^{1/2}} k_I(t, v) T_{\theta}^I(\theta, v), \quad (1)$$

where  $T_{\theta}^I(0, v) = 1$ , and  $k_I(t, v)$  is the Mode I elastodynamic stress intensity factor. The function  $T_{\theta}^I(\theta, v)$  is universal in that it is independent of the overall geometry and the loading. It is of note that the maximum value of  $T_{\theta}^I(\theta, v)$  moves out of the plane  $\theta = 0$  (the plane of crack propagation) as  $v(t)$  increases beyond a certain value. Generally  $k_I(t, v)$  is much more difficult to compute than the corresponding quasi-static stress intensity factor.

The conditions governing crack motion can be expressed in terms of  $k_I(t, v)$  and an experimentally determined critical value that is assumed to be a property of the material. In conventional (quasi-static) LEFM one has  $K_I = K_c$  as the condition for crack instability. In the dynamic generalization of LEFM,  $K_c$  has two counterparts. First, for the initiation of crack growth

$$k_I(t, 0) = K_d(\dot{\sigma}) \quad (2)$$

where  $\dot{\sigma}$  represents the loading rate. For perfectly brittle fracture  $K_c = K_d$ . Similarly, for a propagating crack



$$k_I(t, v) = K_D(v) \quad , \quad (3)$$

where  $K_D$  is known as the dynamic fracture toughness.<sup>1</sup> A third such relation is sometimes used for crack arrest. This is expressed in terms of  $K_I$  and an "arrest toughness" parameter,  $K_a$ . However, while the concept can be useful as an approximation, clearly, for a propagating crack, arrest can only occur when Equation (3) cannot be satisfied. That is, a crack will arrest at a time  $t_a$  when  $k_I < K_D$  for all  $t > t_a$ . Thus, crack arrest is properly viewed as the termination of a general dynamic crack propagation process, not as a unique event governed solely by material properties as suggested by the static crack toughness  $K_a$  approach. While there are circumstances where such a simplistic point of view gives an adequate engineering approximation, its limitations can only be determined via a rigorous, fundamentally correct approach.

Applications of LEFM can be made either in terms of the stress intensity factor or the strain energy release rate parameter  $G$ . The equivalence between these two quantities extends to the dynamic situation as well. Thus, the LEFM criterion for crack propagation can be generalized to the dynamic situation as an equality between  $G$  and its critical value  $R$ , the energy dissipation rate required for crack growth. Then, for given structural geometry, applied loads, and operating temperature, an alternative dynamic crack propagation condition to Equation (3) is

$$G = R(v) \quad . \quad (4)$$

In terms of Equation (4), a dynamic extension of LEFM can be viewed as one which delineates the structural contribution to a propagating crack--the driving force--from the material's fracture property--the resistance. The material property represents the energy dissipated in flow into the crack tip and the fracture processes accompanying crack extension. The crack driving force includes three individual contributions. A net change in these three components, per unit area of crack extension, is called the dynamic energy release rate, or, equivalently, the driving force for crack extension. Formally,

$$G = \frac{1}{b} \left\{ \frac{dW}{da} - \frac{dU}{da} - \frac{dT}{da} \right\} = \frac{1}{bv} \left\{ \frac{dW}{dt} - \frac{dU}{dt} - \frac{dT}{dt} \right\} \quad , \quad (5)$$

where  $U$  is the strain energy,  $T$  is the kinetic energy,  $W$  is the work done on the structure by external loads,  $a$  is the crack length, and  $b$  is the thickness of the body at the crack tip.

Although Equation (5) apparently represents a global quantity that must be evaluated by integrating over the entire structure,  $G$  can always be given a local crack tip interpretation. In particular, it can be directly connected to the dynamic stress intensity factor. For plane strain conditions we have

$$G = \frac{1-v^2}{E} A(v) k_I^2 \quad , \quad (6)$$

<sup>1</sup>In this paper, a crack tip parameter with a letter subscript always denotes a material property. Because the basic definitions are just the same in dynamic LEFM as in conventional static LEFM, there is no reason to put a subscript on the computational quantities as some authors have done, except for I, II, and III to indicate the fracture mode. In fact, because of the confusion between material properties and computed entities that then arises, there is good reason not to do so.

where  $E$  and  $\nu$ , as usual, are the elastic modulus and Poisson's ratio, respectively, and  $A$  is a geometry independent function of the crack speed given by

$$A(v) = \frac{(v/c_T)^2 (1-v^2/c_L^2)^{1/2}}{(1-\nu)[4(1-v^2/c_L^2)^{1/2}(1-v^2/c_T^2)^{1/2} - (2-v^2/c_T^2)^2]} \quad (7)$$

Here  $c_L$  and  $c_T$  are the longitudinal and shear wave speeds, respectively. The function  $A(v)$  is unity at zero crack-tip speed, and increases monotonically to become unbounded as  $v \rightarrow c_R$ , where  $c_R$  is the speed of Rayleigh waves in the material. As  $v \rightarrow c_R$  the elastodynamic stress intensity factor vanishes, and we find  $G \rightarrow 0$  as  $v \rightarrow c_R$ . Consequently, without an external driving mechanism right at the crack tip, cracks cannot propagate faster than Rayleigh surface waves.<sup>2</sup>

Calculational procedures to evaluate  $k$  values are well advanced. Comprehensive review articles are available for further background--see Refs. [1-4]. The experimental determination of dynamic fracture toughness values is also well advanced--see Ref. [5]. More recent information on the application of linear elastic dynamic fracture mechanics to crack arrest predictions are contained in the paper by Hahn, et al in this volume. We now move on to consider solutions giving direct attention to the plastic deformation attending a moving crack tip.

#### Dynamic Strip Yield Plasticity Treatments

The basic postulate of LEFM is that all inelastic irreversible energy dissipation processes that accompany crack extension can be included in a single material property. This property may depend on thickness and temperature, but is independent of the crack length, the applied loads, and the external geometry of the body. This also applies to dynamic fracture mechanics with a dependence on crack speed being allowed to take account of strain rate effects in the intensely deformed region ahead of a propagating crack. This will be valid when the plastic zone at the crack tip is small relative to other dimensions of the cracked body. But, for ductile tough materials, crack tip plasticity can be quite large. For such materials, it becomes necessary to improve on the LEFM autonomous crack tip plastic zone assumption.

In ductile fracture, crack growth takes place by the nucleation and coalescence of voids accompanied by substantial plastic deformation. When the dominant mode of this deformation is shear on  $45^\circ$  planes through-the-thickness, crack growth occurs under conditions which approximate plane stress [6]. A useful model for this situation is the strip yield model given by Dugdale [7] in which the plastic zone is taken to be simply an extension of the crack. In essence, the Dugdale crack model is obtained by superposing two elastic solutions. The first is that for a stress free

<sup>2</sup>The four relations (2), (3), (4), and (6) can be used to establish a theoretical relation between  $K_D$  and  $K_C$

$$K_D = \left[ \frac{1}{A(v)} \frac{R(v)}{R(0)} \right]^{1/2} K_C,$$

where  $R(0)$  corresponds to the static critical energy release rate.



crack of length  $2a$  in an infinite plate under a uniform tension  $\sigma$ . The second is for a crack loaded over intervals of length  $\ell = a - c$  ( $2c$  is the physical crack length) at each of its ends by a tensile stress equal to the yield stress  $Y$  of the material.<sup>3</sup>

Dugdale recognized that the stress singularities in each solution not only occur at the same point but have exactly the same character. The singularities can be made to exactly cancel by adjusting the plastic zone lengths such that

$$\frac{c}{a} = \cos \frac{\pi \sigma}{2Y} \quad (8)$$

A more direct way of deducing Eqn. (8) was given by Goodier and Field [8]. Because the singular terms in the stress functions for each subproblem differ by only a multiplicative constant, the singularities can be abolished by simply setting the coefficient of this term in the combined stress function to zero. In an equally expeditious manner, Goodier and Field were able to determine the normal displacements along the crack line for the Dugdale model. In particular, they found  $\delta$ , the crack opening displacement at the tip of the crack, to be

$$\delta = \frac{(\kappa+1)}{\pi} \frac{Yc}{G} \log \left[ \sec \left( \frac{\pi}{2} \frac{\sigma}{Y} \right) \right] \quad (9)$$

where  $\kappa = (3-\nu)/(1+\nu)$  for plane stress,  $3-4\nu$  for plane strain ( $\nu$  is Poisson's ratio) and  $G$  is the shear modulus. The importance of this result is in the connection that exists between  $\delta$  and other crack tip fracture parameters. As shown by Rice [9]

$$G = J = Y\delta \quad (10)$$

for the Dugdale model. Here,  $G$  is the LEFM energy release rate defined above,  $J$  denotes the value of the  $J$  integral, and  $Y$  is the yield stress. Clearly, the equivalence of the various different parameters arises because the Dugdale model solution is completely within the confines of linear elasticity.

Turning now to rapid crack propagation, Goodier and Field [8] appear to have been first to use the Dugdale model in a dynamic solution. They considered a semi-infinite crack with a finite length strip yield zone propagating at a constant speed in an infinite medium. For this situation, the yield zone length (determined as in the static case by abolishing the singularity) and the crack tip opening displacement are found to be independent of the crack speed. Kanninen, et al [10-11] extended this approach by taking a constant length Dugdale model crack propagating at a constant speed as the basis of a strain rate dependent crack tip plastic zone calculation. In this case, the plastic zone length also is independent of crack speed and, consequently, is just identical to Eqn. (8). The crack tip opening displacement, in contrast, does exhibit a crack speed dependence. This result can be written

<sup>3</sup> It is often incorrectly assumed that plastic deformation like that of the Dugdale model is always obtained if the material is thin enough that plane stress conditions hold. However, 'thinness' is not enough to assure this kind of deformation. Aluminum foil, for example, does not exhibit plastic enclaves of this kind. In addition to the specimen being thin, the material must work harden very little. Then, it will neck as soon as it yields giving through-the-thickness relaxation and, consequently, narrow elongated zones that extend along the prolongation of the crack line.



$$\delta(v) = A(v) \delta(0) \quad , \quad (11)$$

where  $A(v)$  is the function given by Eqn. (7) and  $\delta(0)$  denotes the value of the crack tip opening displacement in the static case; i.e., as given by Eqn. (9).

The idea motivating Kanninen, et al [10,11] was that the strain gradient ahead of a crack tip is so steep that the plastically deformed material is fractured at enormous strain rates. To take this into account, linear superposition was used to obtain a model in which the flow stress varied arbitrarily along the length of the strip yield zone. The flow stress values were assigned in accord with a known strain rate dependent constitutive relation using the crack line displacements in the strip yield zone to determine the local strain rates. In this way, predictions of the limiting speed of ductile crack propagation were made--the limiting speed being governed by the high strain rate dependence of the crack tip flow stress.

Glennie [12,13] has also adopted a model based on a crack propagating with a thin plastic zone and a strain rate dependent yield stress. He similarly concluded that the increased yield stress at high strain rates near the crack tip is the major factor limiting the crack speed. In addition, by comparing with a small-scale yielding calculation, he found that the LEFM stress intensity factor can be used for dynamic behavior even when there is considerable plastic yielding. However, because of the uncertainty of the constitutive relation at high strain rates, no qualitative connection with fracture toughness was made.

While some useful qualitative conclusions can be drawn from these calculations, it is not possible to obtain precise results. There are several shortcomings causing this. First, steady-state crack growth in an infinite medium is an obviously poor approximation to reality. Second, the strain rates that are predicted in these models (and, hence, for which constitutive relations are required) are several orders of magnitude greater than can be measured in any conventional test procedure. Third, this kind of model cannot take account of material unloading in the wake of the crack. Fourth and finally, there is a definite lack of correspondance between a continuum strip yield model and the actual fast fracture mechanisms.

As described by Hoagland, et al [14], the morphology of the plastic deformation attending crack propagation largely consists of highly segmented, but interconnected, regions arising from isolated high toughness zones that are bypassed and remain unbroken even at relatively large distances behind the crack front. They concluded that much of the energy absorbed in unstable crack propagation can be traced to the plastic stretching of these ligaments and, hence, that these are the principal source of the fracture resistance. These experimental observations were supported by computational results obtained with a quasi-static segmented (discontinuous) strip yield zone model. This model is somewhat similar to one proposed by Dvorak [15] to investigate crack growth accompanied by weakened ductile links within a discrete crack zone. This zone, which is supposed to form as a result of selective microcrack propagation in the elastic material ahead of the main crack, is connected with the energy absorption rate and fracture toughness.

All of the models discussed so far in this section suffer from the steady-state assumption which forces the propagating crack to maintain a constant length. A more realistic solution was obtained by Atkinson [16]. He considered a crack expanding at a uniform speed (Broberg model) with collinear strip yield zones, cancelling the singularity to obtain the length of these zones. In contrast to the steady-state solutions, the plastic zone size depends on the crack speed in this problem. In fact, it appears to decrease with crack speed. Unfortunately, the result is quite complicated and

no simple dependence can be extracted. Also, although it appears to be possible, no expression for the crack tip opening displacement was given that could be used to assess the applicability of Eqn. (11).

In short, while quite attractive from an analytical point of view, strip yield models suffer from a number of shortcomings when viewed from a physical mechanisms standpoint. Consequently, other more physically plausible models seem to be required for a realistic assessment of crack tip plasticity in dynamic crack propagation.

#### Dynamic Crack Propagation with Large-Scale Yielding

The strip yield models described in the preceding section were all of the Dugdale type; i.e., with a thin plastic zone ahead of and collinear with the crack. Other types of strip yield models do exist. Models employing dislocation pileups on slip planes inclined to the crack plane to represent crack tip plasticity have been developed. A particularly tractable model employing a superdislocation on an inclined slip plane used by Atkinson and Kanninen [17] has been found to give highly reasonable results for static conditions. Riedel [18] has used a similar picture to address dynamic loading in a strain rate sensitive material. However, it has not so far been possible to incorporate this type of strip yield zone plasticity into a model for a rapidly propagating crack tip.

At present, calculations for unstable crack growth in elastic-plastic materials can only be carried out using numerical methods. There generally are two deficiencies in all these approaches. First, the work is addressed to very specific applications; e.g., pipe fracture. Second, the crack growth criterion used is overly simplistic. Nevertheless, it is useful to briefly review the work that has been done in order to properly assess the directions for future progress. In doing so, attention will be put on the fracture mechanics aspects only. Computational procedures are well covered elsewhere--for example, see Ref. [19].

Current results in elastic-plastic dynamic crack propagation seem to be focused on one of two specific applications: analysis of the Charpy impact test [20-21] and crack propagation in pressurized pipes [22-24]. Ayres [20] performed a two-dimensional (plane strain) elastic-plastic finite element analysis of a precracked Charpy V-notch specimen. Attention was focused on the value of the J integral for the initiation of crack growth. This is given by

$$J = \int_{\Gamma} W dy - T \frac{du}{dx} ds, \quad (12)$$

where  $x$  and  $y$  are Cartesian coordinates ( $y$  normal to the crack line),  $ds$  is an increment of arc length along a contour  $\Gamma$ ,  $T$  is the stress vector on the contour,  $u$  is the displacement vector, and  $W$  is the strain energy density. He further defines a corresponding stress intensity factor  $K_J$  given by

$$K_J = \left[ JE / (1 - \nu^2) \right]^{1/2}. \quad (13)$$

Recognizing that path independence of  $J$  cannot be expected in the dynamic situation, Ayres used a contour that included only the node points closest to the crack tip to minimize the error involved. He concluded that values of  $J$  and  $K_J$  computed in this way offer reasonable candidate criteria for dynamic elastic-plastic fracture.

Norris [21] has taken a different point of view in his analysis of the initiation of crack growth in a Charpy V-notch specimen. He has used a three-dimensional finite difference method and the semi-empirical crack growth criterion developed by Wilkins [25]. Wilkins' criterion stems from



a ductile fracture model and contains material dependent parameters that are adjusted (by trial and error) to fit the experimentally determined fracture initiation in several different geometries. It takes the form

$$D(t) = \int_0^t f(\sigma_m) d\bar{\epsilon}_p \quad (14)$$

with fracture occurring when  $D > D_c$  over a specified region. In Eqn. (14),  $\sigma_m$  denotes the mean stress while  $d\bar{\epsilon}_p$  is the increment of equivalent plastic strain. It can be seen that the parameter  $D$  can be interpreted as an integrated damage with the critical damage being roughly the strain required for crack initiation. In particular, Norris found that fracture initiation in a Charpy test is associated with net section yielding and notch root strains of about 100 percent. In contrast to Ayres, he concludes that a calculation of  $K$  is therefore not relevant.

A complication existing in all realistic dynamic fracture problems is that the boundary conditions are generally time-dependent and, arising from an interaction between the cracked body and the agency supplying the driving force for fracture, they are somewhat uncertain. Thus, in the Charpy specimen, the forces arising from the striking tup must be known in some way. For laboratory tests, the interaction between the test specimen and the loading machine must be taken into account. Lastly, for pipelines, a relation between crack speed and the pressurizing medium must be considered.

In the work of Emery, et al [22], an axial-through-wall crack was suddenly introduced in the wall of a pipe pressurized by either hot water or air. A finite difference solution procedure was used with the leakage of fluid through the crack taken into account. In their elastic-plastic calculations, the crack tip was advanced according to a critical crack tip strain criterion of 2 percent. Of some interest, their results suggest that the simpler models for crack propagation in ductile pipes using essentially LEFM concepts devised by Popelar, et al [26] and by Freund, et al [24] may be reasonably accurate.

The flow area from a postulated pipe break is an important parameter in the design of nuclear reactor steam supply systems. This problem was addressed by Ayres [23]. He determined the largest stable crack that could suddenly appear during normal operating conditions using an elastic-plastic finite element analysis. The  $J$  integral ductile fracture criterion--see Eqn. (12)--was used to predict the stability of the hypothetical cracks.

Some progress has been made on elucidating the effects of crack tip plasticity in dynamic crack propagation, both theoretically [27] and semi-empirically [28]. Broberg [27] concludes from a study of the morphology of material separation that energy dissipation accompanying rapid crack propagation can be separated into two components: that dissipated in a process region in the neighborhood of the crack tip and that dissipated in the plastic region outside the process region. His investigations indicate that the plastic energy dissipation decreases with increasing crack speed while the converse is true for the process zone. The net effect can be a decrease of the total energy dissipation with increasing speed to a minimum followed by a rapid increase at higher speeds. This is consistent with the character of the bulk of the experimentally determined  $K_D = K_D(v)$  curves that have been reported.

In contrast to Broberg's continuum level approach, Shockey, et al [28] have developed a micromechanical computational capability for fast fracture. Their approach is to directly simulate the events occurring in the process zone and, by computing the energy dissipation rates, determine the fracture toughness of a propagating crack. This is done by considering crack growth to occur by the nucleation, growth, and coalescence of micro-fractures in the plastically deforming material at the crack tip. Input is



taken from measurements on specimens fractured under stress wave loads. In this way, they hope to be able to derive fracture toughness values directly from micromechanical flow and fracture processes.

To close this section of the paper, it might be concluded that current analyses are clearly unable to ascribe elastoplastic dynamic crack propagation to a basic condition at the crack tip. Thus, the delineation of a proper fracture criterion may be the most critical outstanding problem in the field. The next section describes recent progress in the static case as a prelude to direct consideration of this problem for dynamic fracture.

### Plastic Fracture Criteria for Stable Crack Growth

While the preponderance of all fracture mechanics applications at the present time are based on LEFM concepts, it is fast becoming clear that there are situations where LEFM-based predictions are so conservative that an inordinate penalty is exacted on the design. An application which has perhaps motivated a more intense research effort than any other is the assessment of the margin of safety of flawed nuclear pressure vessels and piping near and beyond general yielding conditions [29]. A general background on plastic fracture mechanics can be obtained from Ref. [30]. Here, recent work by Kanninen, et al [31,32] on the development of a plastic fracture methodology for stable crack growth under monotonically increasing loading is briefly summarized.

The research reported in Refs. [31,32] proceeds through three main stages. First, laboratory test pieces of pressure vessel steel and of two "toughness-scaled" materials are tested to obtain data on crack growth initiation and stable growth.<sup>4</sup> Next, "generation-phase" analyses are performed. In these, the experimentally observed applied stress versus stable crack growth data are used as input to a finite element model. Critical values for each of a number of candidate crack initiation and stable growth criteria are then generated from the particular test results. Comparison of results obtained for different initial crack sizes and overall test piece geometries provide a basis for an objective appraisal. Finally, in the third stage, "application-phase" finite element analyses are performed. These analyses apply a specific fracture criterion to predict the applied stress versus crack growth behavior for a new set of conditions. The accuracy of the predictions then offers a further basis for appraising the various candidate fracture criteria.

A number of different fracture criteria have been examined. These include the J integral, an elastic-plastic energy release rate  $G^\Delta$ , the crack tip opening angle CTOA, the average crack opening angle COA, and a generalized energy release rate  $G$ . Another is the crack tip force  $F$  which acts at the crack tip nodes in a finite element model during the stable crack growth process.

Computationally, the generalized energy release rate is the sum of two terms. That is,  $G = G^\Delta + G_Z$  where  $G^\Delta$  is the work done in separating the crack faces and  $G_Z$  is the change in the energy contained in the computational process zone (CPZ). Crack growth then proceeds such that

$$R = G \equiv G^\Delta + G_Z \quad (15)$$

<sup>4</sup>Toughness-scaled materials (e.g., aluminum alloys) exhibit essentially the yield/crack growth character of full thickness pressure vessel steel but in reduced thicknesses. This simplifies the testing requirements and thereby allows a wider range of conditions to be examined than would otherwise be possible.

in this approach. Crack instability (fracture) will then occur when  $G > R$  for the prescribed loads or displacements at some crack length. It might be noted that the use of a process zone also circumvents the difficulty associated with a crack tip energy release rate alone. As pointed out by Rice [33],  $G^A$  has a very strong step size dependence, approaching zero in the limit of vanishing crack advance length.

As described in Refs. [31,32], generation-phase computations were made for three aluminum-center cracked panels, an aluminum compact tension specimen, and a steel compact tension specimen. Computational results for the different fracture parameters during stable crack growth show that the quantities that reflect the toughness of the material in the locale of the crack tip-- $G_c$ ,  $R$ ,  $G_{Zc}$ ,  $(CTOA)_c$ , and  $F_c$ --are relatively invariant during stable crack growth. Of these quantities,  $F_c$  and the  $(CTOA)_c$  appear to be most nearly constant. All of the local quantities reflect a loss in crack growth resistance at the beginning of crack extension, but are then constant. In contrast, the parameters that sample large portions of the elastic and plastic strain field-- $(COA)_c$  and  $J_c$ --vary monotonically with stable crack extension. But, within the precision of the analyses, the comparison of the center cracked panel and compact specimen results indicate that only  $(COA)_c$  is independent of geometry. The quantity  $J_c$  shows geometry dependence after a small amount of stable crack growth.

The findings of this research illuminate the basic cause of stable growth in elastic-plastic materials. In the cases analyzed, crack growth stability cannot be attributed to an increase of the toughness of the material. Rather, an increasing load during stable crack growth means that the portion of the energy flow reaching the crack tip region diminishes with crack extension. The reduced energy flow can be thought to result from the "screening" action of the plastic zone accompanying the growing crack in the sense described by Broberg [27].

#### ANALYSIS OF THE NEAR TIP FIELD FOR ANTI-PLANE STRAIN DYNAMIC CRACK PROPAGATION IN AN ELASTIC-PLASTIC MATERIAL

As described in the preceding portion of this paper, crack tip plasticity in dynamic crack propagation has been taken into account in two simplistic ways. The first is by assuming that the plastic region is a small autonomous region controlled by the surrounding elastic stress field. The second is that the plastic deformation is confined to a strip ahead of the crack tip. In both approaches the techniques are essentially those of linear elasticity. Thus, these models do not account for nonlinear elasto-plastic constitutive behavior, nor do they account for different stress-strain paths in loading and unloading. The importance of these effects can only be examined with an incremental plasticity model. Due to the difficulty of performing such calculations, it is appropriate for a preliminary appraisal to consider the anti-plane strain case, and to restrict the attention to the general nature of the near-tip field. An analysis of the near-tip field is presented in this section, by extending a corresponding quasi-static solution given by Amazigo and Hutchinson [35] to the dynamic case.

#### Mode III Crack Propagation

The system of moving coordinates  $(x_1, x_2, x_3)$  shown in Fig. 1 is oriented such that the crack lies in the  $(x_1, x_3)$ -plane,  $x_3$  coincides with the crack front and  $x_1$  is the direction of crack advance. Motion in anti-plane strain is defined by a displacement distribution  $w(x_1, x_2, t)$  where  $w$  is the displacement in the  $x_3$ -direction. Here  $t$  is a time consistent with the moving coordinate system. For future reference we introduce the following notation for material derivatives with respect to time



$$\frac{d(\cdot)}{dt} = (\dot{\cdot}) \quad (16)$$

If the speed of the crack tip is  $v$ , where  $v = v(t)$  is an arbitrary function of time subject to the conditions that  $v(t)$  and  $dv/dt$  are continuous, we have relative to the moving coordinate system

$$(\dot{\cdot}) = \frac{\partial}{\partial t} - v(t) \frac{\partial}{\partial x_1} \quad (17)$$

so that

$$(\ddot{\cdot}) = \frac{\partial^2}{\partial t^2} - \dot{v}(t) \frac{\partial}{\partial x_1} - 2 v(t) \frac{\partial^2}{\partial t \partial x_1} + [v(t)]^2 \frac{\partial^2}{\partial x_1^2} \quad (18)$$

For anti-plane strain the only nonvanishing strain components are  $\epsilon_{13}$  and  $\epsilon_{23}$ . The corresponding stresses are  $\sigma_{13}$  and  $\sigma_{23}$ . The notation can be simplified somewhat by the definitions

$$\tau_i = \sigma_{i3} \quad i = 1, 2 \quad (19)$$

$$\gamma_i = 2\epsilon_{i3} = \frac{\partial w}{\partial x_i} \quad i = 1, 2 \quad (20)$$

Relative to the moving coordinates the equation of motion can then be written as

$$\tau_{i,i} = \rho \ddot{w} \quad (21)$$

where  $i = 1, 2$  and  $\rho$  is the density. Notice that  $\ddot{w}$  follows from Eqn. (18).

The system of governing equations must be completed by constitutive relations. In the following we examine the near-tip fields for the various different constitutive behaviors shown in Fig. 2. First, for linear elasticity--Fig. 2(a)--the constitutive equation is  $\tau_i = G \gamma_i$ . The solution for this case with a near-tip field of the form

$$w = C W(\theta) r^s \quad (22)$$

has been obtained by Achenbach and Bazant [36]. The result is  $s = \frac{1}{2}$  and

$$W(\theta) = \psi_1 \cos \theta - (1-\beta^2)^{\frac{1}{2}} \psi_2 \sin \theta \quad (23)$$

where  $\beta = v/c_T$ ,  $c_T = (G/\rho)^{\frac{1}{2}}$  and

$$\psi_{1,2} = \left\{ \frac{(1-\beta^2 \sin^2 \theta)^{\frac{1}{2}} \pm \cos \theta}{1 - \beta^2 \sin^2 \theta} \right\}^{\frac{1}{2}} \quad (24)$$

Notice that the stresses show the familiar square root singularity of LEFM.

For bilinear elasticity--Fig. 2(b)--an effective shear stress  $\tau$  for a simple shearing history can be defined as

$$\tau = (\tau_1^2 + \tau_2^2)^{\frac{1}{2}} \quad (25)$$

It is assumed that loading and unloading takes place along the same curve. The generalization to anti-plane shear deformation is then

$$\text{and} \quad G \gamma_i = \tau_i \quad \tau < \tau_0 \quad (26)$$



$$G \gamma_i = \tau_i + \left( \frac{G}{G_t} - 1 \right) \left( 1 - \frac{\tau_0}{\tau} \right) \tau_i \quad \tau > \tau_0 \quad (27)$$

For a near-tip solution of the form (22), the strength of the singularity and the general form of  $W(\theta)$  are just the same as for classical elasticity. The relevant elastic constant is, however, the slope of the stress-strain curve in the high strain region,  $G_t$ , since this value applies at the large stresses and strains which pertain at the crack tip. The flattening of the stress-strain curve has important consequences for the significance of dynamic effects in rapid crack propagation. Although the speed of the crack tip may be small as compared to  $(G/\rho)^{1/2}$ , it may be significant compared to  $(G_t/\rho)^{1/2}$ , and it is the latter comparison which counts. In fact, if the fracture process is essentially brittle, the magnitude of  $(G_t/\rho)^{1/2}$  presents an upper limit for the crack propagation speed.

Next we consider small strain nonlinear elasticity which is based on a power law relation of the kind illustrated in Fig. 2(c). For proportional loading the power hardening can be used to represent elastic-plastic material behavior in what is known as the deformation theory of plasticity. For anti-plane shear deformation we have

$$G \gamma_i = \tau_i \quad \tau \leq \tau_0 \quad (28)$$

$$G \gamma_i = \left( \frac{\tau}{\tau_0} \right)^{n-1} \tau_i \quad \tau \geq \tau_0 \quad (29)$$

where  $n > 1$ , and  $\tau$  is defined by Eqn. (25).

For a propagating crack the loading near the crack tip is not proportional; in fact there is a zone of unloading. Thus, deformation theory cannot represent plastic deformation near a rapidly propagating crack tip. Even if Eqn (29) is interpreted as a nonlinear stress-strain relation for an elastic material, it is not possible to obtain a solution of the kind given by Eqn. (22). The reason is that for  $n > 1$  the slope of the stress-strain curve vanishes as  $\gamma_i \rightarrow \infty$ , and the characteristic wave speed becomes zero. Thus, a crack tip moving at any speed is propagating supersonically, and asymptotic solutions of the kind (22) do not apply. On the other hand, if  $n < 1$  the slope of the stress-strain curve becomes unbounded as  $\gamma_i \rightarrow \infty$ . Consequently, the characteristic wave speed of the material becomes unbounded and dynamic effects disappear altogether for a crack tip moving at a bounded velocity.

Finally, we consider rapid crack propagation in strain hardening elastic-plastic materials characterized by  $J_2$  flow theory and a bilinear effective stress-strain curve as shown in Fig. 2(d). For deformation in anti-plane strain the incremental stress-strain relations for loading into the plastic regime ( $d\tau \geq 0$ ) are

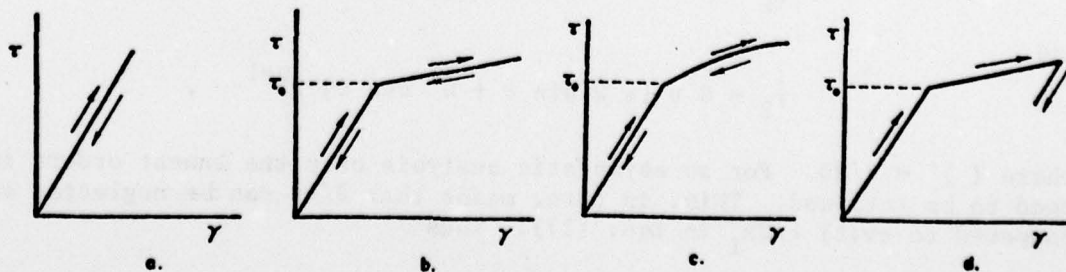


Figure 2

$$G_t d\gamma_i = \alpha d\tau_i + (1-\alpha) \tau_i^{-1} \tau_i d\tau \quad (30)$$

while for elastic unloading ( $d\tau < 0$ )

$$G_t d\gamma_i = \alpha d\tau_i \quad (31)$$

Here,  $\alpha = G_t/G$ . For the problem at hand the stress and strain increments can be replaced by the material derivatives with respect to time, as defined by Eqn. (16). We have

$$G_t \dot{\gamma}_i = \alpha \dot{\tau}_i + (1-\alpha) \tau_i^{-1} \tau_i \dot{\tau} \quad (\dot{\tau} \geq 0) \quad (32)$$

and

$$G_t \dot{\gamma}_i = \alpha \dot{\tau}_i \quad (\dot{\tau} \leq 0) \quad (33)$$

Some useful insight on the influence of strain hardening and unloading on the near-tip fields can be obtained on the basis of Eqns. (32) and (33) by an asymptotic analysis of the near-tip fields. This analysis is carried out in the next section, following the quasi-static treatment of Ref. [35].

#### Dynamic Near-Tip Fields According to $J_2$ Flow Theory

Analogously to Eqn. (22), we seek an asymptotically valid solution for  $\dot{w}$  of the general form

$$\dot{w} = C v \dot{W}(\theta) r^s, \quad (34)$$

where  $C$  is an amplitude factor, while  $\dot{W}(\theta)$  and  $s$  are to be determined. By employing Eqn. (20), the strain rates corresponding to Eqn. (34) are obtained as

$$\dot{\gamma}_i = C v \frac{\partial}{\partial x_i} [\dot{W}(\theta) r^s] \quad i = 1, 2 \quad (35)$$

Now

$$\frac{\partial}{\partial x_1} = \cos \theta \frac{\partial}{\partial r} - \frac{\sin \theta}{r} \frac{\partial}{\partial \theta} \quad (36)$$

and

$$\frac{\partial}{\partial x_2} = \sin \theta \frac{\partial}{\partial r} + \frac{\cos \theta}{r} \frac{\partial}{\partial \theta} \quad (37)$$

so that Eqn. (35) yields

$$\dot{\gamma}_1 = C v [s \dot{W} \cos \theta - \dot{W}' \sin \theta] r^{s-1} \quad (38)$$

and

$$\dot{\gamma}_2 = C v [s \dot{W} \sin \theta + \dot{W}' \cos \theta] r^{s-1}, \quad (39)$$

where  $(\cdot)' = d/d\theta$ . For an asymptotic analysis only the lowest orders in  $r$  need to be retained. This, in turn, means that  $\partial/\partial t$  can be neglected as compared to  $-v(t) \partial/\partial x_1$  in Eqn. (17). Thus

$$(\cdot) \sim -v(t) \frac{\partial}{\partial x_1} \quad (40)$$

Expressions for the stress components can be written as

$$\tau_i = C G T_i r^s \quad i = 1, 2 \quad (41)$$

and

$$\tau = C G T r^s \quad i = 1, 2 \quad , \quad (42)$$

where Eqn. (25) implies

$$T = (T_1^2 + T_2^2)^{1/2} \quad (43)$$

We also define

$$\dot{\tau}_i = C G v \dot{T}_i r^{s-1} \quad (44)$$

and

$$\dot{\tau} = C G v \dot{T} r^{s-1} \quad (45)$$

It follows from (40) and (35) that

$$\dot{T}_i = -s T_i \cos \theta + T' \sin \theta \quad (46)$$

$$T = -s T \cos \theta + T' \sin \theta \quad (47)$$

Now, turning to the equation of motion, substitution of Eqns. (41) and (34) into (21) yields

$$s T_1 \cos \theta - T'_1 \sin \theta + s T_2 \sin \theta + T'_2 \cos \theta = \beta^2 (-s \dot{W} \cos \theta + \dot{W}' \sin \theta) \quad (48)$$

where  $\beta = v/c_T$ , and  $c_T = (G/\rho)^{1/2}$ . Substitution of Eqns. (38) to (45) into (32) yields

$$\alpha (s \dot{W} \cos \theta - \dot{W}' \sin \theta) = \alpha \dot{T}_1 + (1 - \alpha) T^{-1} T_1 \dot{T} \quad (49)$$

and

$$\alpha (s \dot{W} \sin \theta + \dot{W}' \cos \theta) = \alpha \dot{T}_2 + (1 - \alpha) T^{-1} T_2 \dot{T} \quad (50)$$

The corresponding equations for elastic unloading can be obtained by extending the work of Achenbach and Bazant [36]. The solution is

$$\dot{W}_e = (1 - \beta^2 \sin^2 \theta)^{s/2} \cos [s(\omega - \pi)] \quad (51)$$

where  $\tan \omega = (1 - \beta^2)^{1/2} \tan \theta$ . In the sequel we will need  $\dot{W}'_e$ . This takes the form

$$\dot{W}'_e = -s \left\{ \beta^2 \sin \theta \cos \theta \cos [s(\omega - \pi)] + (1 - \beta^2)^{1/2} \sin [s(\omega - \pi)] \right\} (1 - \beta^2 \sin^2 \theta)^{s/2-1} \quad (52)$$

Because of antisymmetry relative to  $\theta = 0$ , only the domain  $0 \leq \theta \leq \pi$  need be considered. The boundary conditions on the crack face and in the plane ahead of the crack tip must reflect a stress-free condition and a condition of antisymmetry, respectively. These can be expressed as

$$\dot{W} = 0 \quad \text{on } \theta = 0 \quad (53)$$

and

$$\dot{W}'_e = 0 \quad \text{on } \theta = \pi \quad (54)$$



As in the quasi-static solution of Ref. [35], the boundary between the loading and unloading zones surrounding the crack tip is assumed to be a radial line emanating from the crack tip at an angle  $\theta = \theta_p$ , see Fig. 3. The field in the loading zone  $0 \leq \theta \leq \theta_p$  is governed by Eqns. (48) - (50). The solutions in the unloading zone  $\theta_p \leq \theta \leq \pi$  are given by Eqns. (51) and (52).

It remains to determine the conditions at the interface  $\theta = \theta_p$ . One condition at  $\theta = \theta_p$  is that  $\dot{r}$  vanishes. This implies that  $\dot{T} = 0$  or, from (47), that

$$-s T \cos \theta + T' \sin \theta = 0 \quad \text{at } \theta = \theta_p \quad (55)$$

In addition, the particle velocity and the stresses must be continuous. Consequently

$$[\dot{W}] = [\dot{W}'] = 0 \quad \text{at } \theta = \theta_p \quad (56)$$

where the following notation has been used

$$[ ] = \lim_{\theta \rightarrow \theta_p^+} ( ) - \lim_{\theta \rightarrow \theta_p^-} ( ) \quad (57)$$

The first of Eqns. (56) is automatically satisfied by writing the solution in the elastic unloading region in the form

$$\dot{W}_e = \dot{W}(\theta_p^-) \dot{W}_e(\theta) \dot{W}_e^{-1}(\theta_p) \quad (58)$$

Here  $\dot{W}(\theta_p^-)$  is the solution in the loading region evaluated at  $\theta = \theta_p$ . It then follows from continuity of  $\dot{W}'$  at  $\theta = \theta_p$  that

$$\dot{W}'(\theta_p^-) = \dot{W}(\theta_p^-) \dot{W}'_e(\theta_p) \dot{W}_e^{-1}(\theta_p) \quad (59)$$

where  $\dot{W}_e(\theta_p)$  and  $\dot{W}'_e(\theta_p)$  follow from Eqns. (51) and (52). Using Eqns. (51) and (52) this can be expressed in the more convenient form

$$\dot{W}' + s \left\{ \frac{\beta^2 \sin \theta \cos \theta}{1 - \beta^2 \sin^2 \theta} + \frac{(1 - \beta^2)^{1/2}}{1 - \beta^2 \sin^2 \theta} \tan [s(\omega - \pi)] \right\} \dot{W} = 0 \quad \text{at } \theta = \theta_p \quad (60)$$

where  $\omega$  is evaluated at  $\theta_p$ . This completes the formulation of the problem.

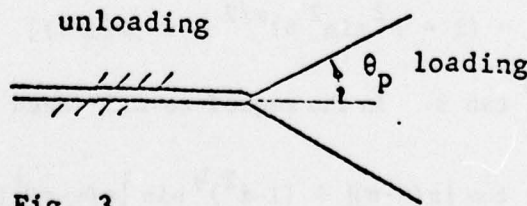


Fig. 3

#### Solution Procedure and Results

The problem has now been reduced to determining a solution for the plastic loading region; i.e., a solution that satisfies the field equations for the region  $0 \leq \theta \leq \theta_p$  given by Eqns. (48) - (50), the boundary condition at  $\theta = 0$  given by (53), and the boundary conditions at  $\theta = \theta_p$  given by Eqns. (55) and (60). The quantities to be determined are  $\dot{W}(\theta)$ ,  $T_1(\theta)$ ,  $T_2(\theta)$ ,  $\theta_p$ , and  $s$ . This is a nonlinear eigenvalue problem which must be solved numerically.

The first step in the numerical procedure is to obtain expressions for the derivatives of the unknown functions. From Eqns. (49) and (50) we have

$$T_1' = -\dot{W}' + \alpha s \dot{W} \cos \theta + \frac{s}{\alpha} T_1 \cot \theta - \frac{1-\alpha}{\alpha} \frac{T_1 T'}{T} \quad (61)$$

and

$$T_2' = \dot{W}' \cot \theta + s \dot{W} + \frac{s}{\alpha} T_2 \cot \theta - \frac{1-\alpha}{\alpha} \frac{T_2 T'}{T}, \quad (62)$$

where (47) has been used to obtain a slight simplification. Next, solving Eqn. (48) and using (61) and (62) gives

$$\begin{aligned} \dot{W}' = -\frac{\sin \theta}{1 - \beta^2 \sin^2 \theta} \left\{ s \beta^2 \dot{W} \cos \theta - \frac{1-\alpha}{\alpha} s T_1 \cos \theta + \frac{s}{\alpha} (\alpha + \cot^2 \theta) T_2 \sin \theta \right. \\ \left. + \frac{1-\alpha}{\alpha} (T_1 \sin \theta - T_2 \cos \theta) \frac{T'}{T} \right\}. \end{aligned} \quad (63)$$

Now, by multiplying (61) by  $T_1$ , (62) by  $T_2$ , and adding the results, making use of Eqn. (43) and the fact that  $TT' = T_1 T_1' + T_2 T_2'$ , gives

$$TT' = \alpha (T_2 \cot \theta - T_1) \dot{W}' + \alpha s (T_2 + T_1 \cot \theta) \dot{W} + s T^2 \cot \theta. \quad (64)$$

By combining Eqns. (63) and (64), a recursion formula can be obtained for  $\dot{W}$ . This is

$$\begin{aligned} \dot{W}' = -s \left\{ T_2 + \beta^2 \dot{W} \sin \theta \cos \theta - (1-\alpha) \left( \frac{T_2 \cos \theta - T_1 \sin \theta}{T} \right) \left( \frac{T_2 \sin \theta + T_1 \cos \theta}{T} \right) \dot{W} \right\} \\ \cdot \left\{ 1 - \beta^2 \sin^2 \theta - (1-\alpha) \left( \frac{T_2 \cos \theta - T_1 \sin \theta}{T} \right)^2 \right\}^{-1}. \end{aligned} \quad (65)$$

The numerical procedure used is simply to replace  $\dot{W}'$  by the difference formula  $[\dot{W}(\theta + \Delta\theta) - \dot{W}(\theta - \Delta\theta)]/2\Delta\theta$  in Eqn. (65). Having a solution at an angle  $\theta$ ,  $\dot{W}(\theta + \Delta\theta)$  can therefore be determined since all quantities on the right-hand side of (65) are known. To obtain  $T_1'$  and  $T_2'$ , Eqn. (64) can be combined with (61) and (62) to get

$$\begin{aligned} T_1' = - \left[ 1 + (1-\alpha)(T_2 \cot \theta - T_1) \frac{T_1}{T^2} \right] \dot{W}' + \\ + s \left[ \cot \theta - (1-\alpha)(T_2 + T_1 \cot \theta) \frac{T_1}{T^2} \right] \dot{W} + s T_1 \cot \theta \end{aligned} \quad (66)$$

and

$$\begin{aligned} T_2' = \left[ \cot \theta - (1-\alpha)(T_2 \cot \theta - T_1) \frac{T_2}{T^2} \right] \dot{W}' + \\ + s \left[ 1 - (1-\alpha)(T_2 + T_1 \cot \theta) \frac{T_2}{T^2} \right] \dot{W} + s T_2 \cot \theta. \end{aligned} \quad (67)$$

Then, replacing  $T_1'$  and  $T_2'$  by similar difference formulas, Eqns (66) and (67) can also be used as recursion relations.

In common with the numerical procedure used by Amazigo and Hutchinson [35], for given values of  $\alpha$  and  $\beta$ , a trial value of  $s$  is selected and values of  $\dot{W}$ ,  $T_1$  and  $T_2$  determined from Eqns. (65) to (67) as functions of  $\theta$ . The



integration proceeds (with a normalization condition that  $T_2(0) = 1$ ) until Eqn. (55) is satisfied. If Eqn. (60) is also satisfied, then  $\theta_p$  has been found and the trial value of  $s$  is the correct one. If not, the estimate of  $s$  is improved and the computation repeated. In this way, the values of  $\theta_p$  and  $s$  given in Tables 1 and 2, respectively, were determined<sup>5</sup>. These data are also shown in Figures 4 and 5. Values of the tangential stress component  $T_\theta$  taken from these results are shown in Figs. 6 and 7.

TABLE 1. CALCULATED VALUES OF  $\theta_p$  FOR DYNAMIC PLASTIC ANTI-PLANE SHEAR CRACK PROPAGATION

$\alpha$	$\theta_p$				
	$\beta=0$	$\beta=0.1$	$\beta=0.25$	$\beta=0.5$	$\beta=0.75$
1.0	1.571	1.576	1.602	1.690	1.786
0.7	1.522	1.528	1.554	1.643	1.731
0.5	1.473	1.478	1.505	1.595	
0.3	1.393	1.398	1.427	1.519	
0.2	1.328	1.334	1.363		
0.1	1.217	1.225	1.259		

TABLE 2. CALCULATED VALUES OF  $s$  FOR DYNAMIC PLASTIC ANTI-PLANE SHEAR CRACK PROPAGATION

$\alpha$	$s$				
	$\beta=0$	$\beta=0.1$	$\beta=0.25$	$\beta=0.5$	$\beta=0.75$
0.1	-0.500	-0.500	-0.500	-0.500	-0.500
0.7	-0.444	-0.444	-0.442	-0.434	-0.396
0.5	-0.395	-0.394	-0.391	-0.375	
0.3	-0.325	-0.324	-0.319	-0.288	
0.2	-0.277	-0.276	-0.269		
0.1	-0.208	-0.206	-0.194		

<sup>5</sup> Results for  $\beta = 0$  given in Tables 1 and 2 are in essential agreement with the quasi-static results given in Ref. [35]. However, it was not possible to verify the results of Ref. [35] for  $\alpha < 0.1$ .

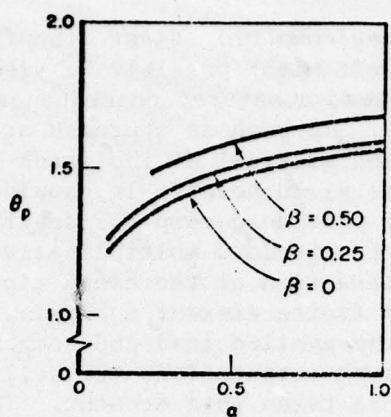


Fig. 4

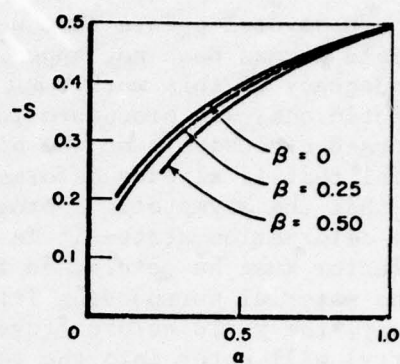


Fig. 5

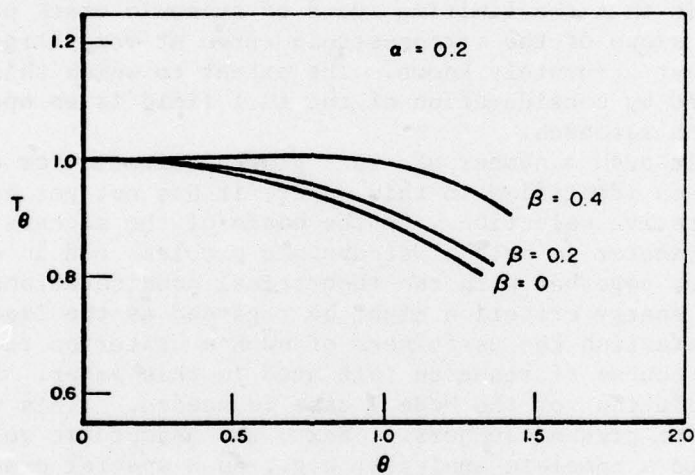


Fig. 6

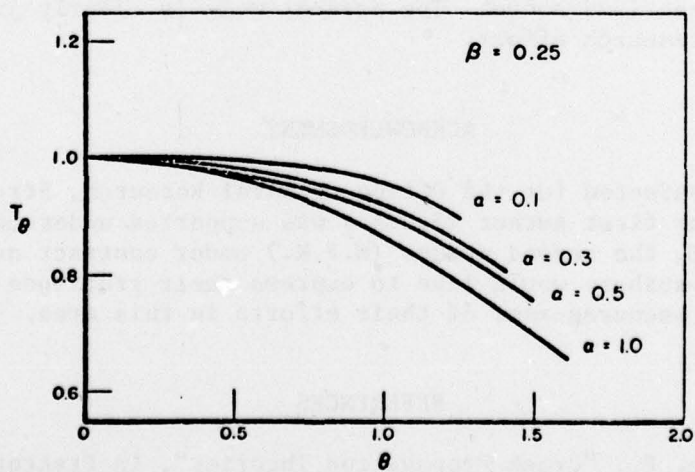


Fig. 7



## DISCUSSION

There are several points that warrant further comment. First, the fact that the yield stress does not appear in the result might possibly be viewed as an inadequacy of this work. But, it is instead a natural consequence of the asymptotic analysis procedure followed here. In such an approach attention is focused exclusively on the highly deformed material at the crack tip; material that is already deformed beyond the yield point. It should also be noted that the asymptotic approach does not provide a complete description of the deformation state--it is known only to within a multiplicative factor. This factor must be determined from the interaction of the crack tip region and the material surrounding it; e.g., in a finite element analysis. In this way, the yield stress (together with the applied load and component geometry) will enter into the result. In such computations the well known effect of strain rate on yielding can then be taken into account. The effect of strain rate on the slope of the stress-strain curve beyond the yield point is less well established. However, it can be seen from the results of this paper that this effect could have a substantial influence on dynamic crack growth.

The preceding comments focus attention on a further key feature of the results. This is that the limiting speed of subsonic crack propagation is dictated by the slope of the stress-strain curve at very large strains where it likely is least accurately known. The extent to which this dominance will be mitigated by consideration of the full field is an open question at this point in the research.

Lastly, although a number of crack growth criteria for dynamic plastic fracture have been identified in this paper, it has not yet been possible to make even a tentative selection. On the basis of the success of the energy release rate parameter in both elastodynamic problems and in quasi-static plastic fracture, together with the theoretical considerations elucidated by Broberg, the energy criterion might be regarded as the leading candidate. To definitely establish the usefulness of such a criterion requires further progress in the course of research initiated in this paper. Specifically, an asymptotic solution for the Mode I case is needed. (This work is already in progress by the present authors.) Next, the asymptotic solution must be incorporated into a complete analysis; e.g., as a special crack tip finite element as in Ref. [37]. The first step is to use experimental results in conjunction with the analysis procedure so devised to appraise various candidate parameters and their formulations. This must be done in both the "generation" and "application" phases as in the quasi-static plastic fracture research described above. The present work is clearly just the first step in such a research effort.

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